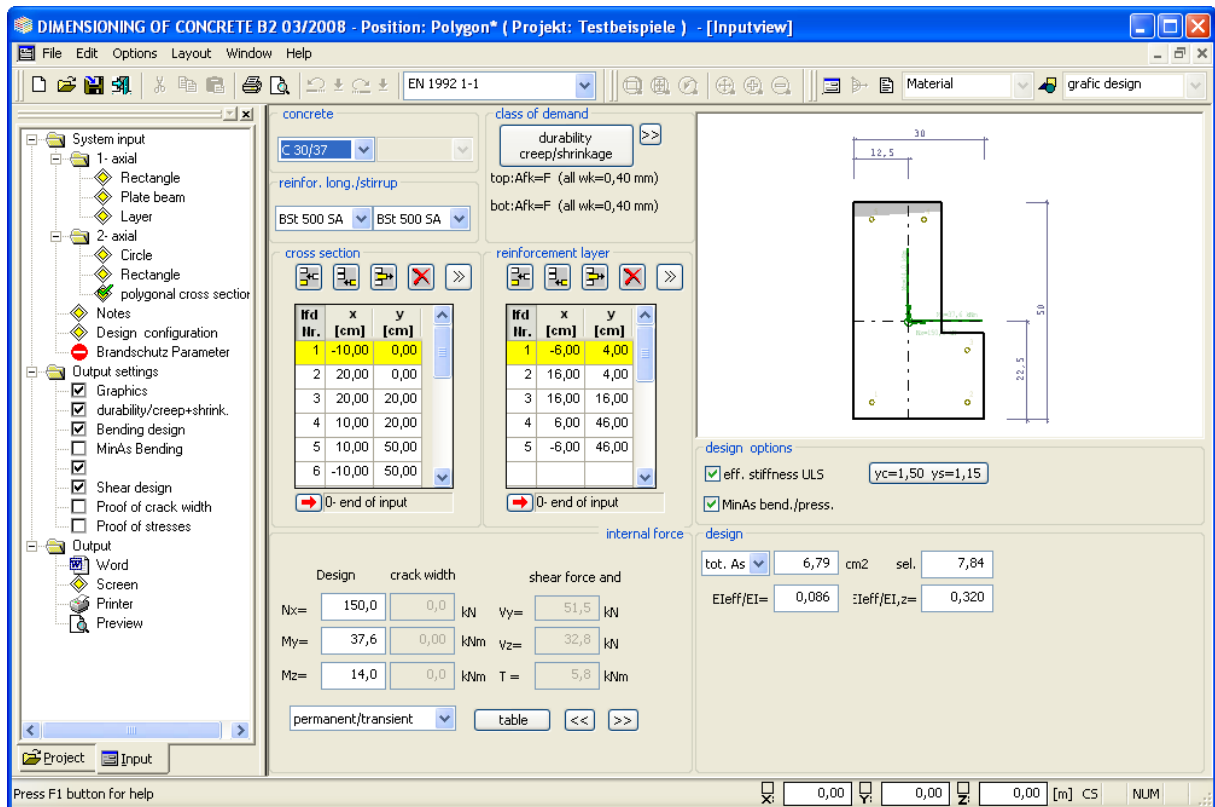


Analyses on Reinforced Concrete Cross Sections

This document includes additional information about our reinforced concrete applications



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Analyses on Reinforced Concrete Cross Sections

Contents

Design for bending and longitudinal force	3
Bases of design.....	3
Design for a given reinforcement ratio	7
Design according to the kd (kh) method	8
Minimum reinforcement for components exposed to bending	10
Minimum reinforcement for compression members.....	10
Lever principle	11
Calculation of the effective rigidity	12
Shear design	15
Shear design according to EN 1992 1-1	15
Serviceability analyses	23
Crack width proof according to EN 1992 1-1.....	23
Stress analysis according to EN 1992 1-1.....	26
Accidental design situation fire	27
Literature	30

See also Standards and terms in the document [B2_eng.pdf](#)

Design for bending and longitudinal force

In the reinforced concrete design, the strain state producing failure is calculated with unknown reinforcement for the given action-effects.

Due to the strain distributions in the ULS defined in the standards, at least one border strain is always known. The internal and external forces must be in balance.

The result are two or, with double bending, three non-linear equations, whereby the internal action-effects are functions of the border strains and the inclination angle of the neutral axis (double bending). The solution is obtained by iteration with the help of the Newton method.

You can select among the kh-(kd)-method (only with uniaxial effect of actions) or the method with given reinforcement ratio for the bending design.

Where cross sections exposed to low action-effects are concerned, the compliance with the minimum reinforcement (compression/bending) can become decisive.

In addition, the application indicates when the permissible maximum reinforcement is exceeded.

Bases of design

	DIN 1045 7/88	Regulation HLB	ÖNORM B4700	EC2 Italy	DIN 1045-1	BS 8110	EN 1992 1-1
Internal action curve of concrete	Figure 11	Fig. R1 + Tab. R7	Figure 7	Figure 4.2	Figure 23	Figure 2.1	Figure 3.3
Maximum stress f_{cd}	β_R acc. to tab. 12	β_R acc. to tab. R7	f_{ck}/γ_c	$\alpha \cdot f_{ck}/\gamma_c$	$\alpha \cdot f_{ck}/\gamma_c$	$0.67 \cdot f_{cu}/\gamma_m$	$\alpha_{cc} \cdot f_{ck}/\gamma_c$
Compressive limit strain concrete ϵ_{cu}	3.5 o/oo	Concrete-depend. tab. R7	3.5 o/oo	3.5 o/oo	Concrete-depend. tab. 9,10	3.5 o/oo	Concrete-depend. tab. 3.1
Compressive strain end of the parabolic area ϵ_{c2}	2 o/oo	Concrete-depend. tab. R7	2 o/oo	2 o/oo	Concrete-depend. tab. 9,10	$0.00024 \cdot \sqrt{(f_{cu}/\gamma_m)}$	Concrete-depend. tab. 3.1
Exponent n	2	Concrete-depend. tab. R7	2	2	Concrete-depend. tab. 9,10	2	Concrete-depend. tab. 3.1
Internal action curve for reinforcing steel	Figure 12	Analogous	Figure 9	Figure 4.5 with $f_{tk}=f_{yk}$	Figure 27	Figure 2.2	Figure 3.8
Stress maximum f_{td}	β_s	β_s	f_{yk}/γ_s	f_{yk}/γ_s	$f_{tk,cal}/\gamma_s$	f_y/γ_m	$K \cdot f_{yk}/\gamma_s$
Limit strain steel ϵ_{ud}	5 o/oo	5 o/oo	20 o/oo acc. to /11/	20 o/oo	25 o/oo	Assumption 10 o/oo	NDP
Strain distrib. ULS	Figure 13	Figure R2	--	Figure 4.11	Figure 30	--	Figure 6.1

The stress-strain curve of the concrete is shown in the parabolic rectangular diagram.

For standard concrete (except BS 8110) with $\epsilon_{c2} = 2$ o/oo and exponent = 2, you can use closed formulas (/2/) to calculate the internal action-effects with rectangular or circular cross sections.

In all other cases (high-performance concrete, T-beams and layers cross sections, concretes acc. to BS 8110) an approximation calculation is performed by splitting the concrete compression zone in thin layers. With cast-in-place complements, the internal actions-effects of the concrete are calculated using the corresponding internal action curves of the different concretes used.

You can optionally take the surface of the concrete displaced by the steel in the compression zone into consideration (→ [Design configuration](#)). The disregard with highly reinforced cross-sections particularly of high-strength concrete common until recently is no longer justified according to /10/ p.13.

DIN 1045 7/88

When performing the design according to DIN 1045 7/88 figure 13, a summary safety coefficient depending on the strains should be taken into consideration.

For high-performance concrete, the area of the rectangle and the parabola as well as the exponent of the function depend on the selected concrete.

DIN 1045-1

In accordance with 10.2.(5), $ed/h < 0.1 \epsilon_{c2}$ with 2.2 ‰ may be assumed with small eccentricities.

This is implemented with the exception of annular and rectangular hollow cross sections and polygonal cross sections. With these cross sections, always ϵ_{c2} acc. to tab. 9,10 is used in the calculation.

According to 10.2.(6) the compressive strain in the plate centre of structured cross sections is to be limited to ϵ_{c2} according to tab. 9,10. This is implemented with the exception of annular and rectangular hollow cross sections and polygonal cross sections.

f _{ck}	characteristic compressive cylinder strength
α	coefficient for the long-term effect and the conversion from compressive cylinder strength to uniaxial compressive strength for standard concrete 0.85 for lightweight concrete 0.75 Note: In case of short-term effects of actions such as in accidental design situations due to load impact (cf. /25/) or earthquake design situations (cf. /23/p. 20-66), you may increase α (0.85 < α ≤ 1.0). → See User-defined concrete
γ _c , γ _s	partial safety coefficients for concrete and steel → Material factors acc. to DIN 1045-1

The inclination of the upper branch of the internal action curve of the reinforcing steel is taken into consideration unless you have unticked this option in the configuration section.

When using high-strength concrete (> C50/60), you should tick the design option "Consideration of the net concrete surface" (cf. /14/ p.161).

ÖNORM B4700

f _{ck}	characteristic fatigue strength (75 % of the characteristic compressive cube strength f _{ck})
γ _c , γ _s	partial safety coefficients for concrete and steel → Material factors acc. to B4700
Minimum moment	acc. to 3.4.3., $M > N \cdot h/10$ applies.

EC2 Italy

f _{ck}	characteristic compressive cylinder strength
α	coefficient for the long-term effect according to 4.2.1.3 (11) α = 0.85
γ _c , γ _s	partial safety coefficients for concrete and steel → See Material factors acc. to EC2

BS 8110

f _{cu}	characteristic compressive cube strength
γ _m	partial safety coefficient of the material

→ See [Material factors acc. to BS 8110](#)

EN 1992 1-1

fck characteristic compressive cylinder strength classes acc. to table 3.1

α_{cc} coefficient for long-term effect NDP

NDP	Standard concrete 3.1.6	Lightweight concrete 11.3.5	Unreinforced 12.3.1
EN	1.0	0.85	0.85
NA_D	0.85	0.75	0.75
NA_GB	0.85	= EN	= EN
NA_A	= EN	= EN	= EN

γ_c partial safety coefficients for concrete NDP

	Permanent/transient 2.4.2.4	Accidental 2.4.3.4	Earthquake
EN	1.5	1.2	1.5
NA_D	= EN	1.3	1.5
NA_GB	= EN	= EN	= EN
NA_A	= EN	= EN	=1.3

possible reduction acc. to Annex A

	A2.1 reduced geometric deviations due to control $\gamma_{c,Red1}$	A2.2 (1) measured or diminished geometric data $\gamma_{c,Red2}$	A2.2 (2) variation coefficient of concrete strength < 10 % $\gamma_{c,Red3}$	A2.3 concrete strength in the mixing plant determines the diminishing factor η ($\gamma_{c,Red*} \eta$)	A2.3 Minimum γ_c ($\gamma_{c,Red4}$)
EN	1.4	1.45	1.35	0.85	1.30
NA_D	1.5	1.5	1.5	0.9	1.35
NA_GB	= EN	= EN	= EN	= EN	= EN
NA_A	= EN	= EN	= EN	= EN	= EN

ϵ_{c2} :

NA_D: with small eccentricities $e_d/h < 0.1$, ϵ_{c2} may be assumed with 2.2 ‰.

This is implemented with the exception of annular and rectangular hollow cross sections and polygonal cross sections. With these cross sections, always ϵ_{c2} acc. to tab. 9,10 is used in the calculation.

ϵ_{cu} :

All: According to 6.1.(5) the compressive strain in the plate centre of structured cross sections is to be limited to ϵ_{c2} according to tab. 3.1. This is implemented with the exception of annular and rectangular hollow cross sections and polygonal cross sections.

f_{yk} characteristic value of the yield point

f_{tk} k · f_{yk} characteristic tensile strength

γ_s : partial safety coefficients for reinforcing steel NDP

	Permanent/transient 2.4.2.4	Accidental 2.4.3.4	Earthquake
EN	1.15	1.0	1.15
NA_D	= EN	= EN	= EN
NA_GB	= EN	= EN	= EN
NA_A	= EN	= EN	=1.0

Possible reduction acc. to Annex A

	A2.1 reduced geometric deviations due to control $\gamma_s, Red1$	A2.2 (1) measured or diminished geometric data $\gamma_c, Red2$
NA_EN	1.10	1.05
NA_D	1.15	1.15
NA_GB	=EN2	=EN2
NA_A	=EN2	=EN2

The inclination of the upper branch of the internal action curve of the reinforcing steel is taken into consideration, unless you have unticked this option in the [Configuration](#).

Minimum moment: $M > N \cdot \max(2 \text{ cm}, h/30)$ applies acc. to 6.1 (4)

Design for a given reinforcement ratio

This function is particularly suitable for the design calculation when compressive force with low eccentricity applies. It can however also be used universally, with multiaxial effects of actions and circular cross sections for instance. The breaking state is assessed by iterative calculation with a given reinforcement layout (biaxial effect of actions) and/or a given ratio of tensile and compression reinforcement (uniaxial effect of actions).

You can reduce the required steel quantity by selecting a particular reinforcement ratio or layout.

Minimum reinforcement

Where compression members ($ed/h < 3.5$) are concerned, the system checks automatically whether a design of the minimum reinforcement will become decisive.

For the design types uniaxial design T-beam, rectangle and layers cross section, the application checks in addition whether the required minimum reinforcement for components affected by bending will become decisive.

For the design types biaxial design rectangular and circular cross sections, the minimum reinforcement is currently not considered.

You can disable the consideration of both minimum reinforcements in the section

→ [Design configuration](#).

DIN 1045 7/88

Booklet 220 of DAfStb¹

Rectangle, uniaxial effect of actions tables 1.10 - 1.12

Rectangle, multiaxial effect of actions tables 1.19 - 1.26

Circle/annulus tables 1.27- 1.30.

Similar design tables for high-performance concrete are included in / 3 /.

DIN 1045-1

Tables for symmetrically reinforced cross sections of standard, high-strength and lightweight concrete acc. to DIN 1045-1 are included in / 10 /.

ÖNORM B4700

Tables for symmetrically reinforced cross sections are included in / 11 /.

EC2 Italy

Tables for symmetrically reinforced cross sections are unknown. Comparisons to / 11 / are possible with restrictions if deviating material parameters are taken into consideration.

BS 8110

Tables for symmetrical reinforced cross sections are included in BS 8110-3, however with $\gamma_s = 1.15$ acc. to BS 8110 (1985).

EN 1992 1-1

NA_D: tables for uniaxial effects of actions in / 46 / ($f_{ck} \leq 50 \text{ N/mm}^2$)
Circular and rectangular cross sections with $d_1/h = 0.05 \dots 0.20$

NA_A: tables for uniaxial effects of actions in / 48 / ($f_{ck} \leq 50 \text{ N/mm}^2$)
Circular and rectangular cross sections with $d_1/h = 0.05 \dots 0.20$

NA_GB: tables for uniaxial effects of actions in / 50 / ($f_{ck} \leq 50$, $f_{ck} = 90 \text{ N/mm}^2$)
Circular and rectangular cross sections with $d_1/h = 0.05 \dots 0.20$

¹ German Committee for Reinforced Concrete

Design according to the kd (kh) method

The kh and kd method are substantially the same. The abbreviation kd also refers to the effective height, height is however abbreviated with d instead of h (DIN 1045-1, EN 1992 1-1).

The method is used for the design on cross sections exposed to uniaxial effects of actions and is the preferable method for bending and longitudinal force with high eccentricity.

$$k_h = \frac{d[\text{cm}]}{\sqrt{\frac{M_s[\text{kNm}]}{b[\text{m}]}}} \text{ is the measure of the effect of actions on the cross section.}$$

In the first place, the layout of a tensile reinforcement is assumed. The resisting moment for a particular strain state is calculated by balancing the moments in regard to the reinforcement layer. The full utilization of the reinforcement produces the strain state with the maximum moment with the compressive limit strain of the concrete on the pressure side and the yield strain at the level of the reinforcement layer. If the applied internal moment is smaller than the limit moment, the breaking state is determined by iterative balancing of the moments and the normal forces. If the applied internal moment is greater than the limit moment, the strain state described above is assumed. The differential moment is balanced with a compression reinforcement.

If compressive strains do not occur, the design is performed according to the lever principle.

In linear elastic calculations of continuous beams, the compression zone height should be limited if no constructive measures are undertaken. Compliance with this criterion is achieved by modifying accordingly the limit steel strain that forms the basis of calculation of a compression reinforcement.

Minimum reinforcement

Where compression members ($ed/h < 3.5$) are concerned, the system checks automatically whether a design of the minimum reinforcement will become decisive.

For the design types uniaxial design T-beam, rectangle and layers cross section, it is checked in addition whether the required minimum reinforcement for components affected by bending will become decisive.

You can disable the consideration of both minimum reinforcements in the section

→ [Design configuration](#).

Particularities with DIN 1045 7/88

The limit strain of steel is assumed with $3 \text{ }^\circ/\text{oo}$.

Compression reinforcement with predominant bending should only be taken into account up to 1 % of the cross sectional area. A compression reinforcement greater than the tension reinforcement is not permitted according to DIN 1045 7/88 (2), i.e. you must apply the method for a given reinforcement ratio.

Corresponding tables are included in Booklet 220 (table 1.a, 1.b)

Particularities with DIN 1045-1

Relative compression zone height in linear elastic calculations of continuous beams:

$$x/d \leq 0.45 \text{ (standard concrete) or } \leq 0.35 \text{ (C55 and higher)}$$

The standard does not limit the compression reinforcement. If the compression reinforcement becomes greater than the tension reinforcement you should however use the method for a given reinforcement ratio in the design.

Tables for cross sections of standard, high-strength and lightweight concrete acc. to DIN 1045-1 are included in / 10 /.

Particularities with EC2 (Italy)

Relative compression zone height in linear elastic calculations of continuous beams:

$$x/d \leq 0.45 \text{ (standard concrete) or } \leq 0.35 \text{ (C55 and higher)}$$

Particularities with B4700

Compression reinforcement results if the steel yield strain can no longer be exploited. You should note in this connection that compression reinforcement must not be taken into account for slabs with $h < 25$ cm according to 3.5.2 (4). You should select a more suitable material or a greater cross section for such effects of actions. Instead of kh ,

$$\gamma = \frac{d}{\sqrt{\frac{M_s}{b \cdot f_{cd}}}}$$
 is used as a measure for the effect of actions on the cross section in the tables.

Tables for cross sections without compressive reinforcement are included in / 12 / tab. 27.

Relative compression zone height in linear elastic calculations of continuous beams with action-effect redistribution:

$$x/d \leq 0.45 \text{ (standard concrete) or } \leq 0.35 \text{ (C55 and higher)}$$

Particularities with BS 810

Relative compression zone height in linear elastic calculations of continuous beams:

$$x/d \leq 0.5 \text{ (see /20/)}$$

Particularities with EN 1992 1-1

Relative compression zone height in linear elastic calculations of continuous beams:

NAD_D $x/d < (1.0 - 0.64) / 0.8 = 0.45$

$f_{ck} > 50 \text{ N/mm}^2$ or lightweight concrete: $x/d < 0.35$

NAD_GB BS: $x/d < (1.0 - 0.40) / 1.0 = 0.6$

$f_{ck} > 50 \text{ N/mm}^2$ $x/d = f(\epsilon_{cu2})$ C90: $x/d = (1 - 0.4) / 1.13 = 0.53$

NAD_A $x/d < (1.0 - 0.44) / 1.25 = 0.45$

$f_{ck} > 50 \text{ N/mm}^2$ $x/d = f(\epsilon_{cu2})$ C90: $x/d = (1 - 0.44) / 1.41 = 0.39$

Minimum reinforcement for components exposed to bending

DIN 1045-1

A minimum reinforcement should be provided for components mainly exposed to bending stress in order to ensure the ductile component behaviour required by 13.1.1.

As per / 29 /, this minimum reinforcement must be considered in combination with the following effects of action:

- Pure bending stress
- Bending with longitudinal pressure as soon as tensile border stresses occur in state I
- Bending with longitudinal tension as soon as compressive border stresses occur in state I

The calculation of the reinforcement is based on the crack moment in accordance with /14/. Longitudinal tensile forces are considered in this connection, the favourable effect of a compressive force is not. An internal lever of $0.9 \cdot d$ is assumed.

EC2 / B4700

With components mainly exposed to bending stress ($e/h > 3.5$), the system checks whether a minimum reinforcement acc. to 5.4.2.1.1 / 3.4.9.4 will become decisive.

BS 8110

With components mainly exposed to tensile stress, the system checks whether a minimum tensile reinforcement acc. to table 3.25 (pure tension, tensile stress on web, tensile stress on flange T-cross section) will become decisive.

EN 1992 1-1

The minimum value of a longitudinal reinforcement exposed to tensile stress is one NDP according to 9.2.1.1.

	Asmin
EN	$= 0.26 \cdot f_{ctm}/f_{yk} \cdot b_t \cdot d > 0.0013 b_t \cdot d$
NA_D	$= (f_{ctm} + N/A_c) \cdot W_c / (f_{yk} \cdot 0.9 \cdot d)$ See /14/
NA_GB	= EN
NA_A	= EN

Minimum reinforcement for compression members

As defined by DIN 1045-1 3.1.19 – compression members are cross sections exposed to compression with a relative load eccentricity in the – ultimate limit state of $e_d/h \leq 3.5$. If biaxial effects of actions apply, compliance with the criterion must be ensured in one direction at least.

DIN 1045 7/88

25.2.2.1 requires a minimum reinforcement for compression members of 0.4 % on the face that is less exposed to compressive stress and of 0.8 % in total referenced to the statically required cross-sectional surface. For walls ($b_0/d_0 > 5$), the minimum reinforcement according to 25.5.5.2 amounts to 0.5 % in total.

For high-performance concrete according to the application guideline, the minimum reinforcement for compression members amounts to 1 % (25.2.2.1). For walls, it can be taken from table R12.

This reinforcement, which is first calculated relative to the real cross section, is used to find the breakage strain state that is needed for the calculation of the resisting longitudinal force.

If it is greater than or equal to the existing longitudinal force, the minimum reinforcement becomes decisive. The reinforcement is reduced according to the ratio between the existing and the resisting longitudinal force because it is referenced to the statically required cross sections.

DIN 1045-1

Columns acc. to 13.5.2: $\text{MinAs} = 0.15 \cdot \text{Nsd}/f_{yd}$
 Walls ($b/h > 4$) acc. to 13.7.1:
 DIN 1045-1 (2001): $\text{MinAs} = 0.0015 \cdot A_c$,
 Slim walls or $\text{NEd} > 0.3 \cdot f_{cd} \cdot A_c$
 $\text{MinAs} = 0.003 \cdot A_c$
 DIN 1045-1 (2008): $\text{MinAs} = 0.15 \cdot \text{Nsd}/f_{yd} > 0.0015 \cdot A_c$
 A_c : concrete cross section

ÖNORM B4700

Columns acc. to 3.4.9.1: $\text{MinAs} = 0.15 \cdot \text{Nsd}/f_{yd} > 0.0028 \cdot A_c$
 Walls ($b/h > 4$) acc. to 5.4.7.2: $\text{MinAs} = 0.0028 \cdot A_c$
 A_c : concrete cross section

EC2 (Italy)

Columns acc. to 5.4.1.2.1: $\text{MinAs} = 0.15 \cdot \text{Nsd}/f_{yd} > 0.003 \cdot A_c$
 Walls ($b/h > 4$) acc. to 5.4.7.2: $\text{MinAs} = 0.004 \cdot A_c$

BS 8110

Acc. to table 3.25 (assumption $\text{Acc}=A_c$) $\text{MinAs} = 0.004 \cdot A_c$

EN 1992 1-1

The minimum reinforcements for columns acc. to 9.5.2 (2) and for walls acc. to 9.6.2 are NDPs.

As,min	Columns	Walls
EN	$= 0.10 \cdot \text{NEd}/f_{yd} > 0.002 \cdot A_c$	$= 0.002 \cdot A_c$
NA_D	$= 0.15 \cdot \text{NEd}/f_{yd}$	$= 0.15 \cdot \text{NEd}/f_{yd} > 0.0015 A_c$
NA_GB	= EN2	= EN2
NA_A	= EN2	= EN2

Lever principle

If the resulting longitudinal tensile force lies in the area of the reinforcement layers, no concrete compression zone results. To simplify the calculation, it is assumed that the reinforcement reaches the yield limit on bottom and on top. The size of the reinforcement then simply depends on the reinforcement distance referenced to the centre of gravity of the cross section and the eccentricity of the resulting force and can be calculated according to the lever principle (DafStb Bl. 220 1.2.8):

See in addition → [Calculation of the effective rigidity](#).

Calculation of the effective rigidity

The state of strain is sought after in which the external and internal action-effects are in balance.

The result are three non-linear equations with three border strains as unknowns. They are resolved by iteration with the help of the Newton method.

The effective rigidity with bending is consequently determined by the strains. The following equations apply

$$Ely_{,eff} = My \cdot H / (\varepsilon_1 - \varepsilon_3) \text{ and}$$

$$Elz_{,eff} = Mz \cdot B / (\varepsilon_1 - \varepsilon_2) .$$

H,B: dimensions of the rectangle enclosing the cross section

ε_1 : strain with maximum pressure

ε_2 : strain in the adjacent corner in x-direction

ε_3 : strain in the adjacent corner in y-direction

Note concerning polygonal cross sections:

With general cross sections, uniaxial effects of actions can also produce curvatures in the direction where the moment is equal to zero.

Therefore, you should consider the curvatures instead of the effective rigidities in the approach to the deformation calculations.

External and internal action-effects

You can select whether the effective rigidity should be calculated in the serviceability limit state (SLS) or ultimate limit state (ULS) (→ see [Design configuration](#)).

The resulting internal action-effects are in accordance with the internal action curves for concrete and steel.

DIN 1045 7/88

Internal action curve of steel	continuously linear behaviour of the steel
Internal action curve of concrete	parabolic rectangular diagram
Action-effects	the summary safety coefficient under service load is 1.0; the summary safety coefficient under breaking load is 1.75.

DIN 1045-1 / EC2 (Italy) / B4700 / BS8100 / EN 1992 1-1

In the serviceability limit state SLS, the material coefficients are set to 1.0, otherwise according to the design situation of the ULS.

Internal action curve of steel	bilinear stress-strain curve
Internal action curve of concrete	ULS: parabolic rectangular diagram SLS: linear internal action curve with E_{cm}
Action-effects	In the serviceability limit state SLS, the design action-effects of the ultimate limit state ULS are divided by a factor defined in the configuration or the action-effects of the quasi-permanent load combination are used → see Configuration .

Particularities with DIN 1045-1

Internal action curve
of steel

If the stress-strain curve is enabled for the calculation of the action-effects, the internal action curve of steel according to figure 26 with rising upper branch applies with

$$f_y = 1.1 \cdot f_{yk} / \gamma_s \text{ and } f_t(\varepsilon_{uk}) = f_y \cdot 1.05 \text{ or } f_y \cdot 1.08 \text{ (}\varepsilon_{uk} \text{ acc. to table 11),}$$

→ see [Configuration](#).

Internal action curve
of concrete

If the stress-strain curve is enabled for the calculation of the action-effects, the internal action curve of concrete according to figure 22 and 8.6.1 (7) applies with

$$f_c = f_{cm} / \gamma_c \text{ and } k = E_{c0} / \gamma_c \cdot \varepsilon_{c1} / f_c$$

(E_{c0} , f_{cm} , ε_{c1} and ε_{c1u} acc. to table 9 and/or 10).

Particularities with EN 1992 1-1

Internal action curve
of steel

According to 5.8.6 (3), a bilinear internal action curve as per figure 3.8 with the design values f_{yd} (yield limit) and $f_{td}(\varepsilon_{ud})$ applies.

Internal action curve
of concrete

If the stress-strain curve is enabled for the calculation of the action-effects (→ see [Configuration](#)), the internal action curve of concrete acc. to figure 3.2 and 5.8.6 (3) applies with

$$f_c = f_{cd} \text{ and } k = E_{cm} / \gamma_{cE} \cdot \varepsilon_{c1} / f_c$$

(E_{cm} , ε_{c1} and ε_{c1u} acc. to tab 3.1 and/or tab. 11.3.1, γ_{cE} is NDP).

	γ_{cE}
EN	1.2
NA_D	1.5
NA_GB	= EN2
NA_A	= EN2

Creep and shrinkage

If creep and shrinkage are enabled in the → [Configuration](#), they are considered in the rigidity calculation as follows:

Creep: If the stress-strain curve of the concrete is non-linear (normally in the ULS), the strain is modified $\varepsilon = \varepsilon / (1 + \varphi)$ for the calculation of the internal action-effects on the concrete.

φ : creep coefficient

DIN 1045-1 $\varphi = \varphi(t_0, \infty)$ acc. to /14/ p.59 ff.

In order to take currently a diminished creep coefficient φ_{eff} acc. to DIN 1045-1 (2008) 8.6.3 (10) into consideration, you must enter it manually

→ see [Environmental conditions/creep coefficient](#).

EN 1992 1-1: $\varphi = \varphi(t_0, \infty)$ acc. to Annex B

In order to take currently a diminished creep coefficient φ_{eff} acc. to 5.8.4 into consideration, you must enter it manually

→ see [Environmental conditions/creep coefficient](#).

With a linear stress-strain curve and in the calculation of curvatures in state 1, the application reduces the modulus of elasticity of the concrete $E_{eff} = E_{cm} / (1 + \varphi)$.

Shrinkage in state I:

Shrinkage is considered via an additional curvature

$1/r_S = \epsilon_{cs} \cdot E_s/E_{eff} \cdot S/I$ in accordance with EC2 Annex 4.

ϵ_{cs} : shrinkage strain

DIN 1045-1: acc. to /14/ p.65 ff.

EN 1992 1-1: acc. to Annex B

S: static moment of the reinforcement relative to the centroid axis (state I) or the neutral axis (state II)

I: moment of inertia of the cross section (state I)

Shrinkage in state II:

According to /24/ p. 18, creep is taken into consideration via a negative compressive prestrain of ϵ_{cs} (creep coefficient acc. to /14/ p. 65 ff.) in the calculation of the internal action-effects on the steel.

Tension stiffening

If the corresponding option is enabled in the → [Configuration](#), the tension stiffening or the contribution of the tensile strength of the concrete between the cracks is considered by modifying the internal action curve of reinforcing steel (cf. /14/ p.35). Depending on the relationship between the steel strain under load in state II and the steel strain under crack action-effects, the steel strain is reduced due to tension stiffening acc. to /14/ figure H.8-3 to ϵ_{sm} .

Component rigidity: only with the cross section types rectangle uniaxial, T-beams and layers cross section.

The distribution coefficient ζ provides for a weighting among the curvatures

in state II $1/r_{II} = (\epsilon_2 - \epsilon_1) / h$ and

and in state I $1/r_I = M / (I_i \cdot E_{eff}) + 1/r_S$

to obtain an average curvature $1/r_m = 1/r_{II} \cdot \zeta + (1-\zeta) \cdot 1/r_I$

$\zeta = \epsilon_{sm} / \epsilon_{s2}$ (cf. /5/ p.292)

ϵ_{sm} : depends on the proportion σ_s/σ_{sr}

ϵ_{s2} : steel strain in state II

σ_{sr} : steel strain in state II under crack action-effects calculated with $f_{ctk0.05}$ (default) or f_{ctm} (option),

→ see [Design configuration](#)

σ_s : steel strain in state II under the load for which the rigidity is calculated (default) or in the infrequent load combination (option),

→ see [Design configuration](#)

$$E_{eff} = My / (1/r_m)$$

Cross-sectional rigidity: the effective rigidity is determined by the curvatures in state II using the factor $k_\zeta = (\epsilon_{sm} - \epsilon_{c2}) / (\epsilon_{s2} - \epsilon_{c2})$ to obtain

$$E_{eff} = M / (k_\zeta \cdot 1/r_{II}) \text{ (cf. /22/ p. 303)}$$

Shear design

Shear design according to EN 1992 1-1

Shear force

The analysis of the shear resistance is based on a truss model with compressive concrete struts and steel ties (stirrups). The minimum stirrup requirements result from the flattest possible strut inclination.

A flatter inclination however reduces the bearing capacity of the struts and increases in addition the forces in the tension chord. The result is an increased shift.

Shear design for vertical shear reinforcement (stirrups):

VEd shear design value (ULS)

VRd,c the shear resistance without reinforcement for the cracked state results from equation 6.2

$$VR_{d,c} = CR_{dc} \cdot \eta_1 \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} \cdot b_w \cdot d \geq VR_{dc} \text{ (Eq. 6.2b)}$$

CR_{dc}: calibration factor acc. to 6.2.2. (1) (NDP)

K₁: empirical strain coefficient

NDP	k ₁ :	CR _{dc}
EN	0.15	0.18/γ _c standard concrete 0.15/γ _c lightweight concrete
NA_D	0.12	0.15/γ _c
NA_GB	0.15,	0.18/γ _c , > C50 test or as C50
NA_A	= EN2	= EN2

η₁ correction factor for lightweight concrete

K = 1 + √(200/d) ≤ 2 [d in mm]

scaling factor, decreases when the effective height increases

ρ_l = A_{sl} / (b_w · d) < 0.02

tensile reinforcement A_{sl} that goes beyond the considered cross section with l_{bd}+d

σ_{cp} = NEd/A_c < 0.2 · f_{cd}

Tension (positive pressure, i.e. higher bearing capacity!)

b_w: lowest cross section width within the effective height

Equation 6.2.b

$$VR_{d,c} > (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$$

NDP	v_{min}
EN	$0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2}$ standard concrete $0.028 \cdot k^{3/2} \cdot f_{ck}^{1/2}$ lightweight concrete
NA_D	$0.0520/\gamma_c \cdot k^{3/2} \cdot f_{ck}^{1/2}$ (d < 600 = EN2 (GK)) $0.0375/\gamma_c \cdot k^{3/2} \cdot f_{ck}^{1/2}$ (d > 800) 0 lightweight concrete
NA_GB	$0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2}$ standard concrete $0.030 \cdot k^{3/2} \cdot f_{ck}^{1/2}$ lightweight concrete (> C50 test or as C50)
NA_A	= EN2

You can optionally perform a calculation in the uncracked state according to 6.4, if the concrete border and main tensile stresses are smaller than $f_{ctk} 0.05/\gamma_c$.

NA_D: does not apply to prestressed element ceilings

Others: applies to single-field systems of prestressed concrete

Components with required shear reinforcement

Cot Θ the design objective is the minimum shear reinforcement, i.e. the flattest possible strut angle (Max Cot Θ) is sought after, at which the bearing capacity of the strut is still ensured.

If torsion stresses apply simultaneously, this bearing capacity can become decisive for the strut angle to be selected.

NDP	Max Cot Θ	Min Cot Θ
EN	2.5	1.0
NA_D	3.0 standard concrete 2.0 lightweight concrete	0.58
NA_GB	= EN2 1.0 with external tension	= EN2
NA_A	1.6 in general 2.5 with overpressure on cross section	= EN2

NAD_D:

$$\text{Cot } \Theta \leq (1.2 - 1.4 \cdot \sigma_{cd}/f_{cd}) / (1 - VR_{d,cc}/VE_d) \quad \text{Eq. 6.7aDE}$$

VR_{d,cc}: crack friction force

$$VR_{d,cc} = \beta_{ct} \cdot 0.1 \cdot f_{ck}^{1/3} \cdot (1 - 1.2 \cdot \sigma_{cd}/f_{cd}) \cdot b_w \cdot z \quad \text{Eq. 6.7.bDE}$$

You can optionally set the strut angle by default (\rightarrow [Design options](#)) to analyze additional sections with the strut angle relevant at the decisive cross section, for instance. This angle must not be flatter than the required one.

z lever arm of the assumed truss model according to the bending design (if unknown, assumption of $0.9 \cdot d$, or of $0.55 \cdot d$ with circular cross sections).

NAD_D: limitation $z < d - 2 \cdot cv,l$ (here cv,l = nomc of the longitudinal reinforcement in the compression zone, acc. to /26/, a limitation of $z < d - cv,l - 3\text{cm}$ applies to $cv,l > 3\text{cm}$).

You can also set a user-defined lever arm by default (→ [Design results](#)).

aswV calculated shear reinforcement acc. to Eq. 6.8

Equation 6.12 is proven through the selection of the strut angle whereby compliance with $VRdmax$ is considered as a criterion.

The application checks whether a minimum shear reinforcement acc. to 9.2.2 (5) for beams or 9.3.1.4 (NAD_D) for slabs will become decisive. The reinforcement is calculated for an average web width (with circular cross sections $bwS = Ac/Da$).

With circular cross sections, an efficiency factor for round stirrups is calculated in accordance with /31/ that increases the required shear reinforcement. The factor takes into consideration that the applying shear force is normally not parallel to the resisting force of the stirrup. Depending on the considered section, the resisting force applies at another angle to the perpendicular.

Min asw/s = $\rho \cdot bw \cdot \sin \alpha$

	ρ (beam) acc. to 9.2.2
EN	$0.08 \cdot \sqrt{f_{ck}/f_{yk}}$
NA_D	$0.16 \cdot f_{ctm}/f_{yk}$
NA_GB	= EN2
NA_A	$0.15 \cdot f_{ctm}/f_{yd}$

NA_A, NA_GB:

slabs ($b/h > 5$): no minimum reinforcement

NAD_D:

slabs with $b/h > 5$ (or if defined so the [Design configuration](#)):

If $VEd < VRdc$, no shear reinforcement is required. Otherwise, a minimum shear reinforcement that is 0.6 times as great as that of beams should be considered.

Junction area $4 < b/h < 5$:

If $VEd < VRdc$, the minimum reinforcement results from interpolation between the zero-fold ($b/h=5$) and the simple value ($b/h=4$), otherwise from interpolation between the 0.6-fold ($b/h=5$) and the simple value ($b/h=4$).

VRd,max The bearing capacity of the struts results acc. to 6.9 and/or equivalent and depends only on $\cot \Theta$. The following equation applies:

$$VRd,max = bw \cdot z \cdot \alpha_{cw} \cdot v_1 \cdot f_{cd} \cdot \cot \Theta / (1 + \cot^2 \Theta)$$

NDP	v_1 acc. to 6.2.2 (6)	Comments
EN	$v_1 = 0.6 \cdot (1 - f_{ck}/250)$ $v_1 = 0.5 \cdot (1 - f_{ck}/250)$ $v_1 = 0.6$ $v_1 = 0.9 - f_{ck}/200 > 0.5$	Eq. 6.6N Eq. 11.6.6N lightweight concrete Eq. 6.10AN $f_{yd} < 0.8 \cdot f_{yk}$ Eq. 6.10bN $f_{yd} < 0.8 \cdot f_{yk}$ and $f_{ck} \geq 60 \text{ N/mm}^2$
NA_D	$v_1 = 0.75 \cdot \eta_1$ $v_1 = 0.75 \cdot \eta_1 \cdot (1.1 - f_{ck}/500)$	$f_{ck} < 60 \text{ N/mm}^2$ $f_{ck} \geq 60 \text{ N/mm}^2$ (IAW DIN, relative to z and with $\gamma_{c'}$)
NA_GB	$v_1 = 0.6 \cdot (1 - f_{ck}/250)$ $v_1 = 0.5 \cdot (1 - f_{ck}/250)$ $v_1 = 0.54 \cdot (1 - \cos \alpha)$ $v_1 = (0.84 - f_{ck}/200) \cdot (1 - \cos \alpha) > 0.5$	Eq. 6.6N Eq. 11.6.6N lightweight concrete $f_{yd} < 0.8 \cdot f_{yk}$ $f_{yd} < 0.8 \cdot f_{yk}$ and $f_{ck} \geq 60 \text{ N/mm}^2$ (> C50 test or as C50)

NDP	α_{cw} acc. to 6.2.2 (6)
EN	Reinforced concrete $\alpha_{cw} = 1.0$ Prestressed concrete $0 < \sigma_{cp} < 0.25 \cdot f_{cd}$: $\alpha_{cw} = 1 + \sigma_{cp} / f_{cd}$ $0.25 < \sigma_{cp} < 0.5 \cdot f_{cd}$: $\alpha_{cw} = 1.25$ $0.5 < \sigma_{cp} < 1.0 \cdot f_{cd}$: $\alpha_{cw} = 2.5 \cdot (1 - \sigma_{cp} / f_{cd})$
NA_D	1.0
NA_GB	= EN2 (> C50 test or as C50)
NA_A	= EN2

The maximum of VRd,max results for a strut angle of 45° .

If VRd,max is smaller than the design value of the shear force, you should increase the cross section or the concrete class.

bw The width bw corresponds with T-beams to the web width b_0 and with layers cross sections to the lowest width in the cross section. Where circular cross sections are concerned, bw corresponds to the lowest width between the resultant compression force and the resultant tension force. If the position of the resultant force is unknown (moment and normal force are equal to zero) a safe distance of the resultant compression force of $Da/40$ is assumed in the calculation.

sl,max maximum stirrup distance acc. to 9.2.2 (6)

	sl,max (NDP acc. to 9.2.2 (6))
EN	$0.75 \cdot d \cdot (1 + \cot \alpha)$
NA_D	distinguished according to shear force utilization with a VRdmax ($\Theta = 40^\circ$)
NA_GB	= EN2
NA_A	$0.75 \cdot d \cdot (1 + \cot \alpha) \leq 250 \text{ mm}$

NAD_D

$VEd < 0.3 \cdot VRdmax$ sMax = $0.7 \cdot h$ beams: < 30 cm (> C50/60: < 20 cm)

$VEd < 0.6 \cdot VRdmax$ sMax = $0.5 \cdot h$ beams: < 30 cm (> C50/60: < 20 cm)

$VEd > 0.6 \cdot VRdmax$ sMax = $0.25 \cdot h$ beams: < 20 cm

VRdmax may be assumed with $\theta = 40$ degrees according to /14/ p. 212

Cast-in-place complement

For cross sections with cast-in-place complement, the bearing capacity of the cast-in-place joint is to be analyzed $vEdi < vRdi$ Eq. 6.23

vEdi shear force to be transmitted per length unit in the joint

$$vEdi = \beta \cdot VEd / (z \cdot bi) \quad \text{Eq. 6.23}$$

VEd: design value of the shear force

z: lever arm of the internal forces,
see shear resistance analysis

NAD_D: if $VRd,c > VEd$, the lever arm limitation with cv can be dispensed with.

β : ratio of normal force in the cast-in-place concrete to total compression force (assumption 1.0)

vRdi design value of the shear force resistance of the joint

$$vRdi = c \cdot \eta_1 \cdot fctd + \mu \cdot \sigma_n + \rho \cdot fy_d \cdot (\mu \cdot \sin \alpha + \cos \alpha) < 0.5 \cdot v \cdot fcd \quad (\text{Eq. 6.25})$$

NAD_D:

$$vRdi = c \cdot \eta_1 \cdot fctd + \mu \cdot \sigma_n + \rho \cdot fy_d \cdot (1,2 \cdot \mu \cdot \sin \alpha + \cos \alpha) < 0.5 \cdot v \cdot fcd$$

σ_n normal stress perpendicular to the joint with $\sigma_{ND} = nEd/bi < 0.6 \cdot fcd$

nEd: design value (pressure: lower, tension: upper) of the normal force perpendicular to the joint per length unit, pressure positive.

bi: effective joint width, reduced total width due to prefabricated formwork, if applicable.

c roughness coefficient according to surface quality

	Very smooth	Smooth	Rough	Interlocked
EN	0.1	0.20	0.40	0.50
NA_D	0	= EN2	= EN2	= EN2
NA_GB	= EN2	= EN2	= EN2	= EN2
NA_A	= EN2	= EN2	= EN2	= EN2

μ Friction coefficient according to surface quality as per table 13

μ	Very smooth	Smooth	Rough	Interlocked
EN	0.5	0.6	0.7	0.9
D	= EN2	= EN2	= EN2	= EN2
GB	= EN2	= EN2	= EN2	= EN2
A	= EN2	= EN2	= EN2	= EN2
I				
PL				

v strength reduction coefficient acc. to 6.2.2 (6)

v	Very smooth	Smooth	Rough	Interlocked
EN	$0.6 \cdot (1-f_{ck}/250)$ 6.2.2 (6)	$0.6 \cdot (1-f_{ck}/250)$ 6.2.2 (6)	$0.6 \cdot (1-f_{ck}/250)$ 6.2.2 (6)	$0.6 \cdot (1-f_{ck}/250)$ 6.2.2 (6)
NA_D	0.0	$0.2 \leq C50$ $0.2 \cdot (1.1-f_{ck}/500)$	$0.5 \leq C50$ $0.5 \cdot (1.1-f_{ck}/500)$	$0.7 \leq C50$ $0.7 \cdot (1.1-f_{ck}/500)$
NA_GB	= EN2	= EN2	= EN2	= EN2
NA_A	= EN2	= EN2	= EN2	= EN2

ρ shear reinforcement percentage of the joint

$$\rho = A_{sw} / A_i = a_{sw} / b_i$$

asw required stirrup reinforcement crossing the joint, hence $vR_{di} = vEd_i$

$vR_{di0} = c \cdot f_{ctd} + \mu \cdot \sigma_n$ bearing capacity without joint reinforcement

$$a_{sw} = b_i \cdot (vEd_i - vR_{di0}) / (f_{yd} \cdot k \cdot \mu \cdot \sin \alpha + \cos \alpha)$$

Torsion

The torsion design is effected via an equivalent hollow cross section. With structured cross sections, only the web cross section is used in the approach by approximation.

The requirement to analyze the torsional resistance instead of the minimum reinforcement results from interaction equations that differ according to the National Annexes.

NAD_A, NAD_GB:

$$T_{Ed}/TR_{dc} + V_{Ed}/VR_{d,c} < 1 \quad \text{Eq. 6.31}$$

T_{Ed}: design value of the torsional moment

TR_{dc}: resisting torsion moment only depending on the tensile strength of the concrete

$$TR_{dc} = W_t \cdot f_{ctd} \quad \text{acc. to /42/ p.290}$$

W_t: section modulus acc. to DAfStb Bl. 220 p. 104

NAD_D:

$$T_{Ed} < V_{Ed} \cdot b_w/4.5 \quad \text{Eq. 6.31aDE}$$

$$V_{Ed} \cdot (1 + (4.5 \cdot T_{Ed}) / (V_{Ed} \cdot b_w)) \leq VR_{dct} \quad \text{Eq. 6.31bDE}$$

Cot θ the design target is the minimum shear reinforcement, i.e. the flattest possible strut angle (Max Cot θ) is sought after, at which the bearing capacity of the strut is still ensured.

This calculation does not automatically produce the reinforcement minimum because the portion of the longitudinal torsion reinforcement increases considerably with flatter struts.

If shear effects of action apply simultaneously, the interaction of shear force and torsion might become decisive for the design.

To simplify the calculation, you can base the torsion analysis exclusively on the assumption $\text{Cot } \theta = 1.0$ (45 degrees) (see [Design configuration](#)).

NAD_D:

Calculation of the strut angle acc. to /51/, p. 173 ff

$$\text{Cot } \theta \leq (1.2 - 1.4 \cdot \sigma_{cd}/f_{cd}) / (1 - \text{VRd,cc}/\text{VEd, T+V}) \text{ acc. to Eq. 6.7.aDE}$$

VEd, T+V: resultant effect of actions

$$\text{VEd, T+V} = \text{VEd, T} + \text{VEd, V} \cdot \text{teff, l} / b_w$$

VEd, V: effect of actions due to shear force

VEd, T: effect of actions due to torsion

$$\text{VEd, T} = \text{Ted} \cdot z_i / (2 \cdot A)$$

VRd, cc: crack friction force acc. to Eq. 6.7.bDE

$$\text{VRd, cc} = \beta_{ct} \cdot 0.1 \cdot f_{ck}^{1/3} \cdot (1 - 1.2 \cdot \sigma_{cd}/f_{cd}) \cdot \text{tef, i} \cdot z$$

TRd,max design value of the resisting torsional moment acc. to Eq. 6.30 or equivalent depending exclusively on cot θ . The following equation applies:

$$\text{TRd, max} = 2 \cdot v \cdot \alpha_{cw} \cdot f_{cd} \cdot A_k \cdot \text{tef, l} \cdot \cot \theta (1 + \cot^2 \theta)$$

tef, i: effective wall thickness

$$\text{tef, l} = A / U$$

< 2 · d1 double distance of reinforcement

< ba real wall thickness with hollow cross sections

Ak: surface enclosed by the wall centre lines

α_{cw} : coefficient analogous to VRd,max

The maximum for TRd,max results for a strut angle of 45 degrees. If TRd,max is smaller than the design value of the torsional moment, you should increase the cross section of the concrete class.

aswT the required stirrup reinforcement due to torsion results from

$$\text{aswT}^* = \text{TEd} / (2 \cdot A_k \cdot f_{yd} \cdot \cot \theta) \quad /46/ \text{ p. 283, for instance}$$

Because only one leg of torsion stirrups may be considered in the calculation, $\text{aswT} = 2 \cdot \text{aswT}^*$ applies.

The minimum shear reinforcement becomes decisive if $\text{aswV} + \text{aswT} < \text{aswMin}$ is true.

AsL additional longitudinal reinforcement due to torsion

$$\text{Asl} = \text{TEd} \cdot \cot \theta \cdot U_k / (2 \cdot A_k \cdot f_{yd}) \quad \text{Eq. 6.28}$$

Uk: circumference of area Ak

With combined shear force and torsional effect of actions, the following interaction condition must be complied with:

$$T_{Ed}/T_{Rd,max} + V_{Ed}/V_{Rd,max} < 1 \quad \text{Eq. 6.29}$$

NAD_D:

For compact cross section applies

$$(T_{Ed}/T_{Rd,max})^2 + (V_{Ed}/V_{Rd,max})^2 < 1 \quad \text{Eq. 6.29aDE}$$

The stirrup cross section results from $as_w(V+T) = as_wV + as_wT$.

Serviceability analyses

Crack width proof according to EN 1992 1-1

Based on the **crack formula Eq. 7.8** the maximum limit diameter still in compliance with the permissible crack width is calculated for an external effect of actions that depends on the decisive combination of actions and for a selected reinforcement.

$$w_k = s_{r,max} \cdot (\epsilon_{sm} - \epsilon_{cm})$$

Decisive action-effect combinations and permissible crack width acc. to table 7.1 (NDP)

The considered NAs all require the proof of a permissible crack width of 0.3 mm for reinforced concrete components of exposure class XC2 and higher.

The proof for XC1 is based on a crack width of 0.4 mm for aesthetical reasons (exception NA_GB: 0.3 mm)

The decisive load combination is the quasi-permanent one (Qk).

Due to the fact that the tensioning bars are highly susceptible to corrosion, prestressed concrete components have to comply with higher requirements in regard to the load combinations (infrequent (Sk), frequent (Hk)) and the permissible crack width to be proven. In some cases, a proof of decompression (Dek.) might be required for particular load combinations.

The regulations may differ in the national annexes.

Post-tensioned concrete:

	X0, XC1	XC2/XC4	XS1-3, XD1-3
EN	0.2 + Hk	0.2+ Hk and Dek. Qk	Dek. Hk
NA_D	0.2 + Hk	0.2+ Hk and Dek. Qk	0.2+ Hk and Dek. Qk
NA_GB	0.2+ Hk	0.2+ Hk and Dek. Qk	Dek. Hk
NA_A	0.2+ Hk	0.2+ Hk and Dek. Qk	0.2+ Hk and Dek. Qk

Pre-tensioned concrete:

	X0, XC1	XC2/XC4	XS1-3, XD1-3
EN	0.2 + Hk	0.2+ Hk and Dek. Qk	Dek. Hk
NA_D	0.2 + Hk	0.2+ Hk and Dek. Qk	0.2+ Sk and Dek. Hk
NA_GB	0.2+ Hk	0.2+ Hk and Dek. Qk	Dek. Hk
NA_A	0.2+ Hk	0.2+ Hk and Dek. Qk	0.2+ Sk and Dek. Hk

The crack width results from the maximum crack spacing s_{rmax} and the average strain difference $\epsilon_{sm} - \epsilon_{cm}$ of concrete and steel.

$\varepsilon_{sm} - \varepsilon_{cm}$: **average strain difference between steel and concrete (Eq. 7.9)**

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} (1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0,6 \frac{\sigma_s}{E_s}$$

k_t : 0.6 short-term action
0.4 long-term action

σ_s : steel strain in state II
Calculation with $E_{ceff} = E_{cm} / (1 + \varphi (t=UE))$

$\alpha_e = E_s / E_{cm}$

ρ_{eff} : reinforcement percentage in the effective tension zone

$$\rho_{eff} = (A_s + A_p \cdot \xi^2) / A_{ceff}$$

A_s : reinforcing steel surface included in A_{ceff}

A_p : tensioning steel surface included in A_{ceff}

ξ : factor for the bond characteristics of tensioning steel

A_{ceff} : surface of the effective tension zone

$$A_{ceff} = h_{eff} \cdot b_{eff}$$

h_{eff} 2.5 · D1 < (h-X0II)/2
X0II: compression zone height in state II:

b_{eff} effective tension zone width
not included in the standard, but in /11/ for instance
 $b_{eff} = 0.5 \cdot b_{eff}(Z.I) + 2 \cdot c_1$
(cf item Cur_D / Cur_D_BP)

Sr,max: **maximum crack spacing:**

$$s_{r,max} = k_3 \cdot c + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi}{\rho_{p,eff}}$$

k_1 : coefficient reinforcement bond characteristics
0.8 good bond characteristics
1.6 poor bond characteristics

k_2 : coefficient of the strain distribution
Bending: 0.5
Tension 1.0
Bending + tension $(\varepsilon_1 + \varepsilon_2) / (2 \cdot \varepsilon_1)$

k_3 : coefficient for concrete cover
 c : concrete cover on longitudinal reinforcement
 k_4 : coefficient
 ϕ : average diameter of the tensile reinforcement

NDP	K3	K4
EN	3.4	0.425
NA_D	0	$1/(3,6 \cdot k_1 \cdot k_2) < \phi \cdot \sigma_s / (3,6 \cdot f_{ct,eff})$
NA_GB	= EN2	= EN2
NA_A	0	$1/(3,6 \cdot k_1 \cdot k_2) < \phi \cdot \sigma_s / (3,6 \cdot f_{ct,eff})$

The limit diameter ϕ is obtained by rearranging the crack formula.

Compared with table 7.2, more favourable (larger) limit diameters may result because the simplifications the table is based on are dispensed with.

If the resultant limit diameter cannot be realized, you should increase the selected reinforcement.

Minimum reinforcement due to indirect action:

The application calculates a minimum reinforcement acc. to 7.3.2 for imposed bending on top and bottom if the corresponding option was enabled in the dialog [Control of crack width proof](#).

The minimum reinforcement for T-beams is calculated separately for the web and the flange, whereby the rectangle over the total cross section height is considered as the web and the remaining parts of the slab as the flange. You can take different bar diameters for flange and web into account.

$$A_{s,min} \cdot \sigma_s = k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} \quad (\text{Eq. 7.1})$$

k coefficient for internal indirect action
1.0 ($h \leq 300$ mm)... 0.65 ($h \geq 800$ mm)
h: lower value of the partial cross section

$f_{ct,eff}$ tensile strength, f_{ctm} ($t \leq 28d$)
NA_D: ≥ 2.9 N/mm² when $t \geq 28d$

k_c coefficient for the stress distribution
 $k_c = 0.4 \cdot (1 - \sigma_c / (k_1 \cdot f_{ct,eff} \cdot h/h'))$

σ_c : concrete stress (state I) under crack action-effects
in the centre of gravity of the partial cross section

Flanges box, T-cross sections, for crack action-effects compl. under tension

$$k_c = 0.9 \cdot F_{cr} / (A_{ct} \cdot f_{ct,eff}) \geq 0.5$$

F_{cr} : tensile force in the flange under crack action-effects (state I)

σ_s : calculated according to modified diameter D_{s1} for compliance with permissible w_k

EN2: Tab. 7.2N formula and derivation, see /12/

$$D_{s1} = D_s \cdot f_{ct0} / f_{ct,eff} \cdot 2 \cdot (h-d) / (k_c \cdot h_{cr})$$

NA_D: Tab. 7.2DE formula see /13/, p. 196 ff.

$$D_{s1} = f_{ct0} / f_{ct,eff} \cdot 4 \cdot (h-d) / (k \cdot k_c \cdot h_{cr}) < D_s \cdot f_{ct0} / f_{ct,eff}$$

NA_A: Tab. 5 analogous to NA_D

Stress analysis according to EN 1992 1-1

Concrete, infrequent combination

$$\sigma_c < k_1 \cdot f_{ck} \quad k_1 = 0.6 \text{ (all considered NAs)}$$

The objective is to prevent the destruction of the concrete structure. Alternatively, you can increase the concrete cover or enclose the compression zone with reinforcement.

EN2	recommended with the exposure classes XD, XS or XF.
D	can be dispensed with where unprestressed components in typical building construction are concerned if the percentage of redistribution < 15 %.

Concrete, quasi-permanent combination

$$\sigma_c < k_2 \cdot f_{ck} \quad k_2 = 0.45 \text{ (all considered NAs)}$$

When this limit value is exceeded, linear creep can no longer be assumed. If applicable, an increased creep coefficient according to Eq. 3.7 should be considered.

Reinforcing steel, infrequent combination

$$\sigma_s < k_3 \cdot f_{yk} \quad k_3 = 0.8 \text{ (all considered NAs)}$$

Whereas the crack width proof for reinforced concrete becomes only decisive under the quasi-permanent combination, yielding of the reinforcement should also be prevented under the infrequent combination.

With indirect action:

$$\sigma_s < k_4 \cdot f_{yk} \quad k_4 = 1.0 \text{ (all considered NAs)}$$

Calculation of the existing stresses

The calculation of the steel stresses should be performed with a reduced modulus of elasticity

$$E_{\text{eff}} = E_{\text{cm}} / (1 + \varphi(t_0, \infty)) \text{ acc. to /11/}.$$

This takes the long-term behaviour of concrete into consideration. The concrete withdraws from its contribution in the bearing of the effects of actions by creep i.e. redistribution to the reinforcing steel.

Acc. to /11/, this can often be neglected where compact cross sections are concerned. With T-beams however, the resultant steel stresses increase by 5 % in comparison to a calculation that does not consider the creep coefficient. A corresponding note as in ENV 1992 1-1 para. 4.4.1.3 (3) is however missing in EN 1992 1-1.

Correspondingly early times are decisive for the calculation of the concrete stresses, i.e. $\varphi = 0$ in this case.

Accidental design situation fire

Fundamental considerations

According to MLTB 9/2007², the analysis could be performed using a simplified calculation method in the analysis according to DIN ENV 1992 1-2:1997.

In the meantime, the new Eurocode DIN EN 1992 1-2:2006(/42/) and a draft of the NA (/44/) have been published.

Current publications refer to the new Eurocode such as the lectures of Dr. Müller at the Seminar of the Bavarian Chamber of Engineers 2007, the lectures of Dr. Richter concerning the hot model columns method held at the occasion of the 12th Solid Construction Seminar 2008 in Munich and the articles by Prof. Quast and Dr. Richter in "Beton- und Stahlbetonbau 2/2008" (/41/).

The latter article states that the results of the simplified calculation acc. to EN 1992 1-2 Annex B.3 differ only slightly from those of the general method. In addition, the authors found out that this is not the case when the method according to Annex B.2 (zone method) is used.

The hot design and the rigidity calculation in this application are also based on the simplified method B.3. In accordance with the recommendation in /41/, the thermal strains are also considered in this connection.

The first version of this application allows only analyses for rectangular and circular cross sections with fire attack on four sides. We therefore have dispensed with a thermal analysis and decided to use the temperature profiles according to Annex A of the Eurocode instead.

As the exact position of the steel is decisive for the result, the additional module "Polygonal design" B2-Poly should be available. The analyses in the event of fire are performed with the cross section types "rectangle with general point reinforcement" and "circle with general point reinforcement".

Temperature profiles

The temperature profiles in /42/ Annex A are based on the following assumptions:

- Fire attack on four sides according to the standard temperature-time curve (ETK)

- Specific heat acc. to 3.3.2

- Humidity of 1.5 %

- Thermal conductivity λ_c acc. to 3.3.3 with lower limit value

- Convective heat-transfer coefficient $\alpha_c = 25 \text{ W}/(\text{m}^2 \text{ K})$

- Circular cross section $D = 300 \text{ mm}$

- Squared cross section $h = 300 \text{ mm}$

- Fire resistance classes R30, 60, 90 120

With deviating cross-sectional dimensions, a uniform spacing of the temperature isolines from the outer edge is assumed. This means that with larger cross sections, the temperatures ($h > 30 \text{ cm}$) are slightly higher, i.e. they are on the safe side.

Whereas, with smaller cross sections, the temperatures ($h < 30 \text{ cm}$) are slightly lower and decrease progressively the smaller the cross section and the higher the fire resistance is.

We therefore recommend to perform the analysis with a temperature addition of 20 - 40 degrees.

For the fire-resistance R180, no temperature profiles are specified in Annex A. In the case of rectangular cross sections, temperature profiles according to CEB Bulletin 145 (/45/) implying temperatures on the safe side are used.

Since corresponding temperature profiles for circular cross sections have not been published yet in any literature known to us, we have based them on our own FEM calculations.

² Sample List of Technical Construction Regulations

External action-effects

The action-effects of the combination for the accidental design situation fire should be used.

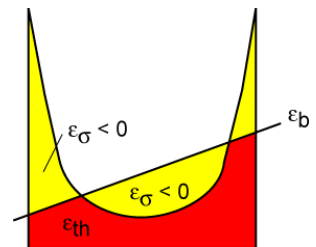
Internal action-effects

In order to calculate the internal action-effects on the concrete, the concrete cross section is divided into elements with an edge length of 1 cm. The internal action-effects on the element result with the stress-strain curves corresponding to the average element temperatures acc. to /42/ figure 3.1 and table 3.1. Calcerous aggregates can be taken into consideration, if applicable. The thermal strain results according to figure 3.5.

According to the recommendation in /41/ the stress-strain curves of /43/ table 8 are used for high-strength concretes. The thermal strains result according to /43/ figure 37. The use of high-strength concrete currently still requires coordination with the site supervision.

The internal action-effects on the reinforcing steel depend on the temperatures in the reinforcement points acc. to /42/ figure 3.3 and table 3.2. The more favourable behaviour of hot-rolled steel can be taken into consideration in this connection, if applicable. According to /44/ steel of class X requires a proof by experimental testing and is therefore currently not supported. The thermal strain results according to /42/ figure 3.

The stress-generating strain ε_{σ} in an arbitrary point of the cross section results from the thermal strain ε_{th} depending on the temperature and the bending strain ε_b in this point. The equation $\varepsilon_{\sigma} = \varepsilon_b - \varepsilon_{th}$ applies.



For the concrete results a typical bearing behaviour whereby a smaller outer ring due to the considerably diminished stress-strain curve with high temperatures and an inner area with $\varepsilon_{\sigma} > 0$ (tension) withdraw from the contribution to bear the action-effects.

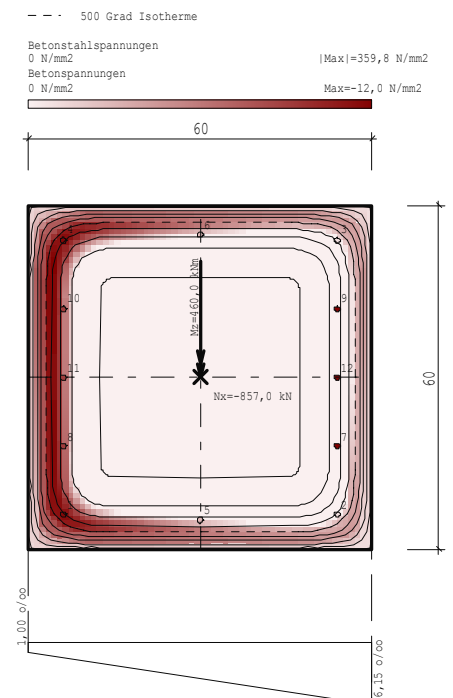
The internal action-effects on the reinforcing steel react very sensible to the location of the reinforcement point, a minor change in position of 1 cm produces a measurable change in the steel strain.

Design

The strain state (bending plane) at which the internal and external action-effects are in balance is sought after by iterative approximation.

The internal action-effects on the steel are first calculated for a reinforcement area still unknown whereby a uniform weighting of the entered reinforcement points is assumed.

The strain plane is varied between the defined breakage strains. The required reinforcement quantity results directly from the resultant strain state.



Calculation of the effective rigidity

The strain state (bending plane) at which the internal and external action-effects are in balance is sought after by iterative approximation.

The internal-action effects on the steel are calculated for a selected reinforcement that can vary from point to point.

The effective rigidity results from the found strain state. The following applies:

$\text{effEI}_z = M_z \cdot h / (\varepsilon_1 - \varepsilon_2)$ and $\text{effEI}_y = M_y \cdot h / (\varepsilon_1 - \varepsilon_3)$, whereby $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are the corner strains describing the bending level at the rectangle enclosing the cross section.

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